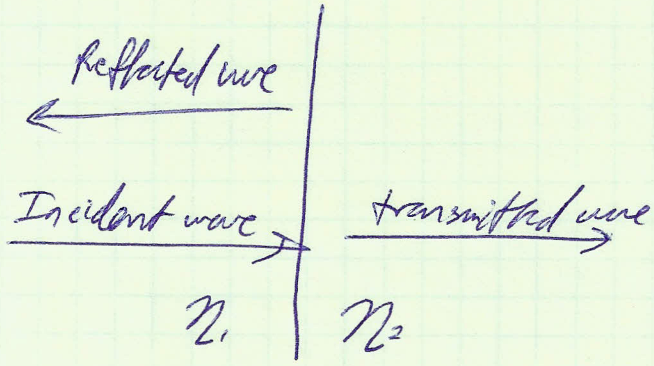
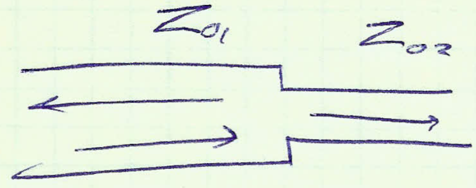
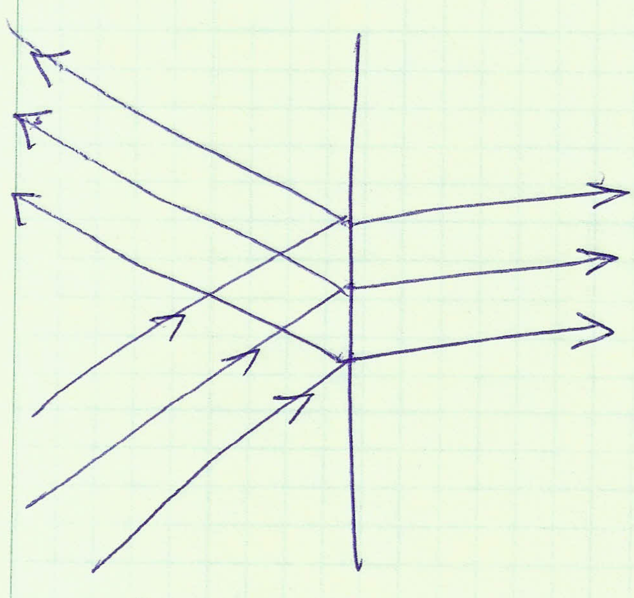
Transmission and reflection at normal incidence



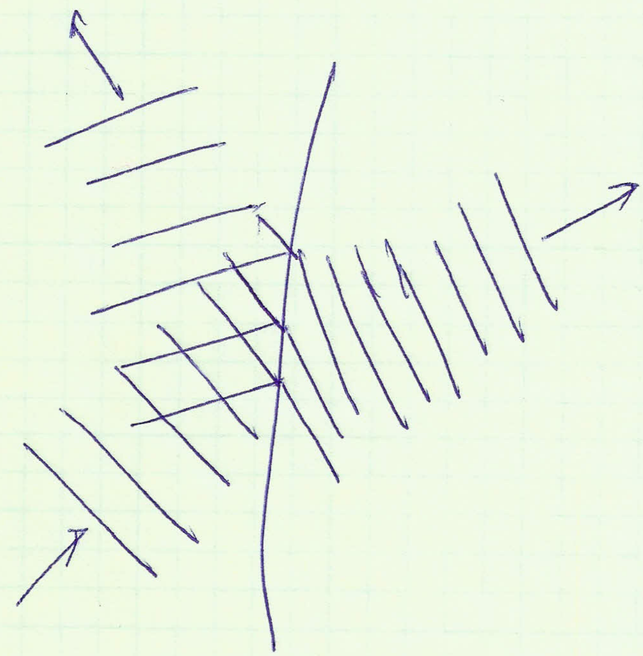


transmission line analogy

Bending between two media



Ray representation



wavefront representation

- Rays must travel the same optical distance
- Wavefronts must line up at interface (phase matching)

Boundary between lossless media

$$\tilde{E}^i(z) = \hat{x} E_0^i e^{-jk_1 z}$$

$$\tilde{H}^i(z) = \hat{z} \times \frac{\tilde{E}^i(z)}{\eta_1} = \hat{y} \frac{E_0^i}{\eta_1} e^{-jk_1 z}$$

} incident

$$\tilde{E}^r(z) = \hat{x} E_0^r e^{jk_1 z}$$

$$\tilde{H}^r(z) = (-\hat{z}) \times \frac{\tilde{E}^r(z)}{\eta_1} = -\hat{y} \frac{E_0^r}{\eta_1} e^{jk_1 z}$$

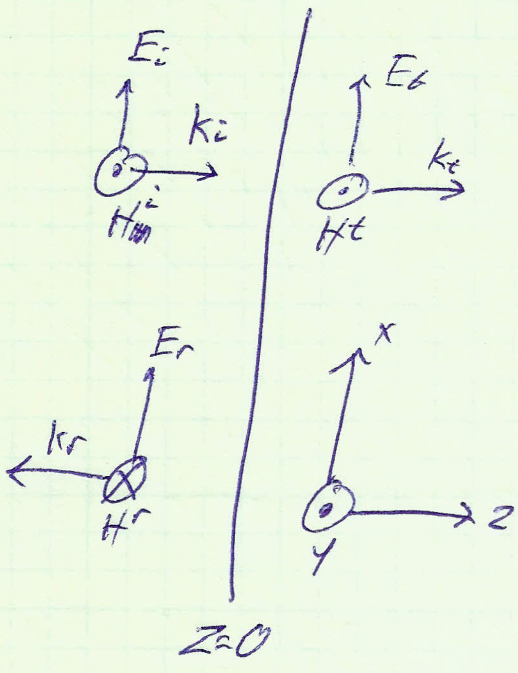
} transmitted

$$\tilde{E}^t(z) = \hat{x} E_0^t e^{-jk_2 z}$$

$$\tilde{H}^t(z) = \hat{z} \times \frac{\tilde{E}^t(z)}{\eta_2} = \hat{y} \frac{E_0^t}{\eta_2} e^{-jk_2 z}$$

} reflected

- E and H must be continuous across interface



$$E_0^i + E_0^r = E_0^t$$

$$H_0^i - H_0^r = H_0^t$$

$$\rightarrow \frac{E_0^i}{\eta_1} - \frac{E_0^r}{\eta_1} = \frac{E_0^t}{\eta_2}$$

solve for E_0^r and E_0^t:

$$E_0^r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_0^i = \Gamma E_0^i$$

$$E_0^t = \frac{2\eta_2}{\eta_2 + \eta_1} E_0^i = \tau E_0^i$$

$$\tau = 1 + \Gamma$$

for nonmagnetic media use $\eta = \frac{\eta_0}{\sqrt{\epsilon_r}}$

Power flow in lossless media

$$S_{av_1}(z) = \frac{1}{2}\mathrm{Re}\left[\tilde{E}_1(z) \times \tilde{H}_1^*(z)\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\hat{x} E_0^i \left(e^{-jk_1 z} + \Gamma e^{jk_1 z}\right) \times \hat{y} \frac{E_0^{i*}}{\eta_1}\left(e^{jk_1 z} - \Gamma^* e^{-jk_1 z}\right)\right]$$

$$= \hat{z}\frac{|E_0^i|^2}{2\eta_1}\left(1 - |\Gamma|^2\right)$$

$$\vec{S}_{av_1} = \vec{S}_{av}^i + \vec{S}_{av}^r$$

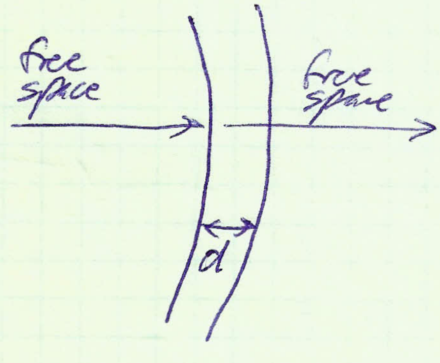
$$S_{av_2}(z) = \frac{1}{2}\mathrm{Re}\left[\tilde{E}_2(z) \times \tilde{H}_2^*(z)\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\hat{x}\tau E_0^i e^{-jk_2 z} \times \hat{y}\tau^* \frac{E_0^{i*}}{\eta_2} e^{jk_2 z}\right]$$

$$= \hat{z}|\tau|^2 \frac{|E_0^i|^2}{2\eta_2}$$

$$\frac{\tau^2}{\eta_2} = \frac{1-\Gamma^2}{\eta_1}$$ from previous definition of τ and Γ

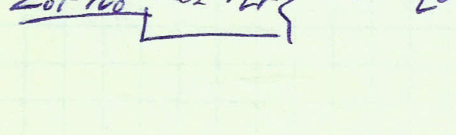
Example: Radome design



remember from chapter 2:

1/2 wavelength delay has no effect on reflection coefficient

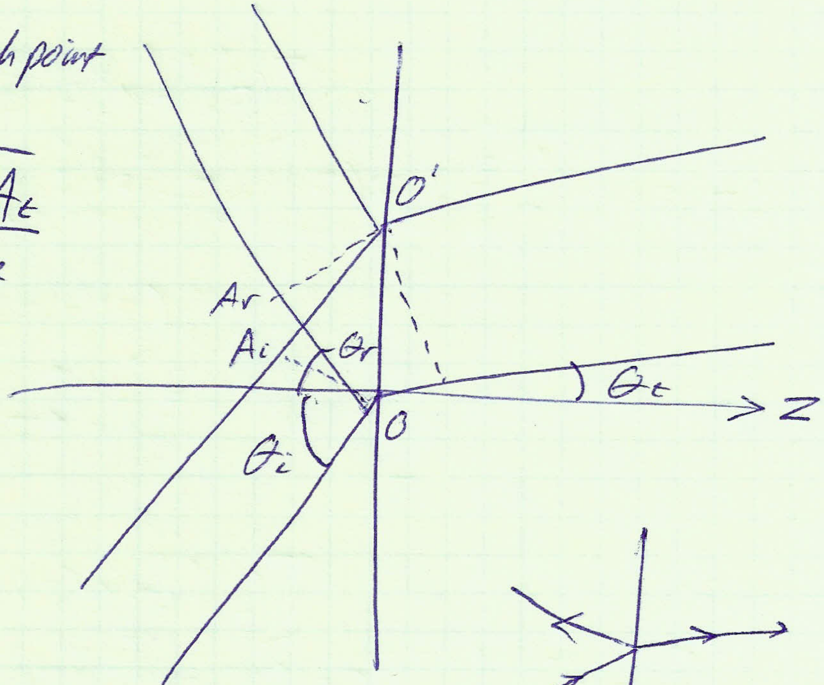
$$d = \frac{n\lambda_2}{2} = \frac{n\lambda_0}{2\sqrt{\epsilon_2}}$$



Snell's Laws

same time to travel to each point

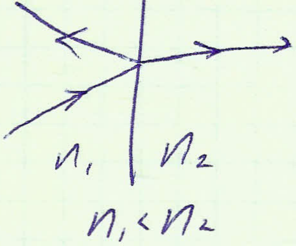
$$\frac{\overline{A_iO'}}{u_{p1}} = \frac{\overline{OA_r}}{u_{p1}} = \frac{\overline{OA_t}}{u_{p2}}$$



$\overline{A_iO'} = \overline{OO'} \sin\theta_i$

$\overline{OA_r} = \overline{OO'} \sin\theta_r$

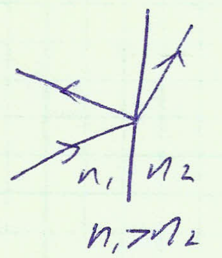
$\overline{OA_t} = \overline{OO'} \sin\theta_t$



Snell's law of reflection:

$$\theta_i = \theta_r$$

Snell's law of refraction:



$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}} = \frac{n_1}{n_2} = \frac{\eta_2}{\eta_1}$$

$\mu_1 = \mu_2$

for non magnetic media

Optical fibers

$\sin\theta_c = \frac{n_c}{n_f} \rightarrow \sin\theta_3 \geq \frac{n_c}{n_f}$

$\cos\theta_2 \geq \frac{n_c}{n_f} \quad (\cos\theta_2 = \sin\theta_3)$

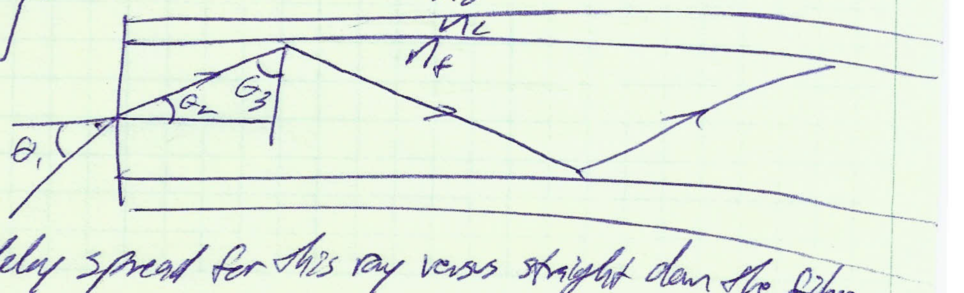
critical angle

$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\sin\theta_2 = \frac{n_0}{n_f}\sin\theta_i$

$\cos\theta_2 = \left[1 - \left(\frac{n_0}{n_f}\right)^2 \sin^2\theta_i\right]^{1/2}$

$\sin\theta_i \leq \frac{(n_f^2 - n_c^2)^{1/2}}{n_0}$



- note that there will be a delay spread for this ray versus straight down the fiber.